Evaluation of the Seismic Demand Chord Rotations of Structural Reinforced Concrete Members

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Abstract

In the present article, the calculation of seismic demand chord rotations (elastic and plastic) of structural reinforced concrete (r/c) members, without using various inelastic springs at their ends, is analytically developed. Particularly, in order to calculate, in the nonlinear area, the seismic demand chord rotations of structural r/c members, the following three cases must be examined separately: (1) Exterior roof joint of single-storey frame, (2) interior roof joint of single-storey frame and (3) exterior & interior floor joint of multi-storey frame. The above-mentioned three cases consist the theoretical base for the calculation of the seismic demand chord rotations of r/c members of each other case. This presentation is very useful for the develop of suitable software about the seismic nonlinear (static/dynamic) structural analysis without using nonlinear springs.

Keywords

Seismic Demand Chord Rotations; Seismic Nonlinear Static Analysis; Seismic Nonlinear Response History Analysis

Introduction

The point of calculating the seismic demand chord rotations (elastic ones and plastic ones) of the structural r/c members, without using springs at their ends, is always addressed in timelines. In the past, many techniques have been presented about the simulation of r/c structures in nonlinear seismic analysis, i.e. see Banon *et al* (1981) and their references, but in this case, we are going to focus on the contemporary Eurocode EN 1998-3 (2005). This Seismic Code proposes a flexural nonlinear law of Moment-Chord Rotation at each critical end-section of each r/c member of the structure. On an equivalent note, we can say that each structural r/c member is based on the seismic behavior of a cantilever for each critical end-section. In the past, many papers have been published, of varying accuracy as regards their methodology, with reference to the calculation of seismic demand plastic rotations of r/c structural members (Kanaan and Powell, 1975; Litton, 1975; Gupta and Krawinkler, 1999; Goel and Chopra 2004; Eom *et al*, 2012). However, in the present article we are going to look at the same issue from a different point of view, without the use of inelastic springs. In order to estimate the seismic demand chord rotations of structural r/c members at yielding and ultimate state, without inelastic springs, the geometrical base combined with the Mechanic Principles are used and this is the main target of this article. Additional theoretical bases (Makarios, 2012; Makarios, 2013) and numerical results of examples took place in the past in other works for verification reasons (Makarios, 2005; Makarios, 2009; Makarios, 2010; Makarios & Asteris, 2012;).

Theoretical Background

The available yielding chord rotation of a structural r/c member (in real of an r/c cantilever) can be estimated by the following semi-empirical equation that is proposed by Eq.(A.10a)/sect.A.3.2.4 of Eurocode EN 1998-3, and is based on work by Panagiotakos and Fardis (1999, 2001), where all possible sources that contribute to the yielding rotations of the end-section, such as the action of bending moment and the shear force and the extraction or lap-splice slip of longitudinal steel bars from the fixed-base (or the joint) of the cantilever, taking into account.

$$\theta_{\mathrm{y}} = \frac{\varphi_{\mathrm{y}} \cdot \left(L_{\mathrm{s}} + a_{\mathrm{V}} \cdot z\right)}{3} + 0.00135 \cdot \left(1 + \frac{1.50 \cdot h}{L_{\mathrm{s}}}\right) + \frac{\varepsilon_{\mathrm{y}} \cdot d_{\mathrm{b}} \cdot \left(f_{\mathrm{ym}}/\mathrm{CF}\right)}{6 \cdot \left(d - d_1\right)\sqrt{f_{\mathrm{cm}}/\mathrm{CF}}} \qquad (1)$$

where a_{V} is zero, when the flexural failure precedes the shear failure, and a_{V} is one, when the shear failure precedes the flexural one; z is the length of the internal lever arm, considered equal to $d - d_2$ in beams and

columns, while d and d_2 are the depths of the tension and compression reinforcement from the external compressive fiber of the section, respectively. In addition, d_1 is the distance from the tension reinforcement to the external tension fiber of the section, h is the depth of the geometric section of the member and ε_y is the yielding steel strain that is considered equal to $\varepsilon_y = f_{ym}/(CF \cdot E_s)$. Moreover, E_s is the Elasticity Modulus of the steel, d_b is the mean diameter of the steel bars of longitudinal reinforcement in meters and f_{ym} and f_{cm} are the yielding stresses (mean value in MPa) of steel and concrete, respectively. Finally, CF is the Confidence Factor according to Table 3.1 of EN 1998-3, while it is worth noting that, the first part of Eq.(1) is the contribution of the flexural deformations. The second part of Eq.(1) is the contribution of the shear deformations (by experimental data) and the third part is the contribution of the extraction or lap-splice slip of the longitudinal steel bars from the fixed-base of the cantilever, using experimental data also.

Consider that the above-mentioned cantilever possesses constant geometric dimensions along its length and, also, has a linear-elastic behavior until the critical section at its base reaches the yielding state. Thus, it can be concluded that the flexural section-stiffness EI of the cantilever section can be constant for the total length of the member and thus its effective value EI_{eff} can be calculated by Eq.(2), having known the corrected value of yielding rotation θ_y by Eq.(1), where E is the Elasticity Modulus of the concrete and I_{eff} is the effective moment of inertia of the member cross-section. Hence, the effective flexural stiffness EI_{eff} is given according to sect.A.3.2.4(5)/ EN 1998-3:

$$EI_{eff} = \frac{M_y \cdot L_s}{3 \cdot \theta_y} \tag{2}$$

Therefore, in the case of a real structural member (column or beam) that has plastic hinges at its two ends, the mean effective flexural stiffness EI_{eff} of the member-section can be estimated as the arithmetic mean of four different bend states, at the two ends of the element, for a positive and negative sign of the flexural moments. This mean effective flexural stiffness ($EI_{eff,mean} = EI_{eff}$) of the member cross-section is suitable for the rational modelling in the case of dynamic cyclic behavior, when the building is subjected to earthquake loading. It should be noted that, the above-mentioned assumption about the EI_{eff} is quite rational, in the case where two plastic hinges are presented simultaneously at the two ends of a structural member.

Analytic Estimation of Seismic Demand Plastic Chord Rotations of Structural Members

Exterior Roof Joint of Single-Storey Frame

Consider column ij with a height H_{ij} (FIG.1), which possesses a section effective flexural stiffness $EI_{ij,eff}$, constant along its whole length. Moreover, the beam possesses section effective flexural stiffness $EI_{b,eff}$, where E is the Modulus of Elasticity of the concrete, and $I_{ij,eff}$, $I_{b,eff}$, which are the effective moments of inertia of the column and beam cross-section, respectively. Moreover, we consider that the section effective flexural stiffness $EI_{b,eff}$ and $EI_{ij,eff}$ is calculated by Eq.(2). In addition, we consider that the beam is loaded with the permanent gravity loading p, while the horizontal displacement u_j is gradually increased (due to pushover analysis) and points (o) and (e) are the null-moment points on the column and beam, respectively. It is worthy to note that the position of these two points is altered in each step of the nonlinear analysis, while the yielding flexural moment M_{yb} at the beam-end is known from the section analysis using the known fiber elements (XTRACT, 2007).

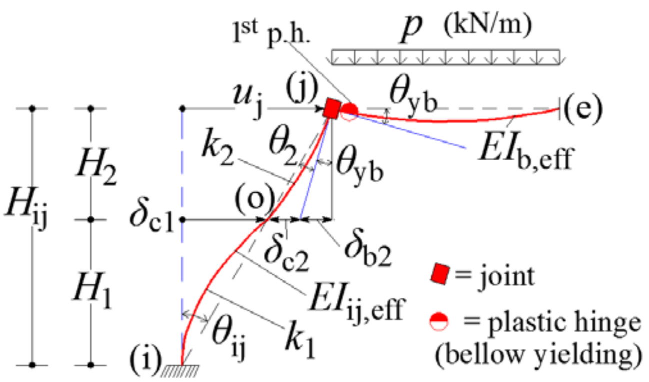

FIG. 1 GEOMETRICAL ELEMENTS FOR ESTIMATION OF THE SEISMIC DEMAND BEAM YIELDING CHORD ROTATION

1) Calculation of Seismic Demand Beam Yielding Chord Rotation

Consider the horizontal relative displacement $u_j = u_{y1}$ regarding the appearance of the first plastic hinge at

the beam-end (FIG. 1). Then, the flexural moment at the top of the column is M_{yb}, because the equilibrium of the joint is true. Therefore, the seismic demand column chord rotation θ_{ij} at the column bottom, which in the single-storey structures is equal with the demand "floor angular deformation", is directly given by:

$$\theta_{ij} = \frac{u_j}{H_{ij}} \tag{3}$$

If moment M_i at the column bottom-section is less than respective yielding column moment M_{yc}, namely $M_i < M_{yc}$, no plastic hinge appears and, thus, the seismic demand column chord rotation θ_{ij} at the column bottom is less than the respective available column chord rotation θ_{yc} that resulted from Eq.(1).

It is worth noting that in this case, the plastic hinge at the column top cannot appear, because there is a plastic hinge at the beam-end. Thus, the seismic demand column chord rotation θ_2 is always less than the respective available column chord rotation θ_{yc} at the column top, as the later is calculating by Eq.(1). According to the geometry in FIG.(1), the seismic demand column chord rotation θ_2 is always calculated by:

$$\theta_2 = \frac{\delta_{c2}}{H_2} \tag{4}$$

where $H_2 = M_{yb}/Q_c$, $\delta_{c2} = Q_c/k_2$ and k_2 is the lateral stiffness of the "ideal cantilever j-o" that is given by:

$$k_2 = \frac{3EI_{ij,eff}}{H_2^3} \tag{5}$$

Therefore, the seismic demand beam yielding rotation θ_{yb} of the beam-end is given as a function of the horizontal relative yielding displacement u_j of the single-storey frame (FIG.1):

$$\theta_{yb} = \theta_{ij} - \theta_2 = \frac{u_j}{H_{ij}} - \frac{\delta_{c2}}{H_2} \tag{6}$$

2) *Calculation of Seismic Demand Beam Plastic Chord Rotation*

Consider an advanced analysis step, where the horizontal relative displacement u_{dem} is greater than the horizontal relative yielding displacement u_{y1} (FIG.2). Then, the seismic demand column chord rotation $\theta_{ij'}$ at the column bottom-section is:

$$\theta_{ij'} = \frac{u_{dem}}{H_{ij}} \tag{7}$$

To calculate the seismic demand plastic rotation θ_{pb} in the first plastic hinge at the beam-end, the following two separate cases must be examined:

- (i) Case: Before the appearance of the second plastic hinge at the column bottom, where $M_i \le M_{yc}$ and <!--[if !vml]--> $\theta_{1j'} \le \theta_{yc}$ it is true;
- (ii) Case: After the appearance of the second plastic hinge at the column bottom, where $M_i = M_{yc}$ and $\theta_{ij'} \ge \theta_{yc}$ it is true;

With reference to (i) Case, we consider that after the appearance of the first plastic hinge at the beam-end, the existing column moment is $M_{i'} \le M_{yc}$ and the respective column chord rotation is $\theta_{ij'} \le \theta_{yc}$ (FIG.2). Therefore, the chord rotation $\theta_{2'}$ at the top of the column is calculated geometrically:

$$\theta_{2'} = \frac{\delta_{c2'}}{H_{2'}} \tag{8}$$

where $Q_{c'}$ is the current shear force of ij column at this step of analysis, $H_{2'} = M_{yb}/Q_{c'}$, $\delta_{c2'} = Q_{c'}/k_{2'}$ and $k_{2'} = 3EI_{ij,eff}\big/H_{2'}^3$.

Therefore, the total (yielding plus plastic) beam chord rotation $\theta_{b,tot}$ at the joint (j), as a function of the seismic demand horizontal relative displacement u_{dem}, is geometrically given by:

$$\theta_{b,tot} = \theta_{ij'} - \theta_{2'} = \frac{u_{dem}}{H_{ij}} - \frac{\delta_{c2'}}{H_{2'}} \tag{9}$$

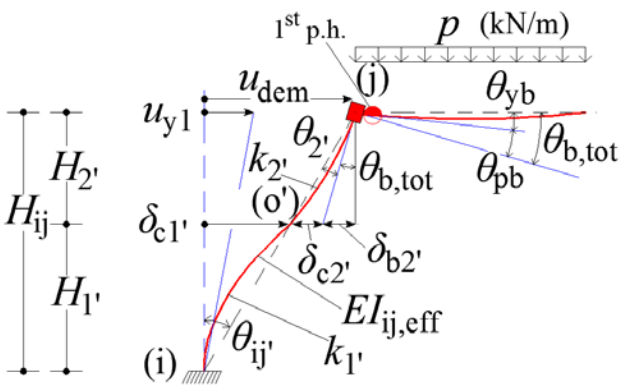

FIG. 2 ESTIMATION OF SEISMIC DEMAND BEAM PLASTIC CHORD ROTATION

Therefore, as it resulted by FIG.(2), the seismic demand beam plastic rotation θ_{pb} that is developed in the first plastic hinge of the beam-end is directly given by:

$$\theta_{pb} = \theta_{b,tot} - \theta_{yb} \quad \Rightarrow$$

$$\theta_{pb} = \left(\frac{u_{dem}}{H_{ij}} - \frac{\delta_{c2'}}{H_{2'}}\right) - \left(\frac{u_{y1}}{H_{ij}} - \frac{\delta_{c2}}{H_2}\right) \tag{10}$$

or

$$\theta_{pb} = \frac{u_{dem} - u_{y1}}{H_{ij}} - \frac{\delta_{c2'}}{H_{2'}} + \frac{\delta_{c2}}{H_2} \tag{11}$$

Afterwards, for the seismic performance target of "Significant Damage" level that is defined by EN 1998-3, if $\left(\theta_{yb} + \theta_{pb}\right) \le 0.75\theta_{ub}$ is true for the examined structural member, then the seismic demand is satisfied. Also, for the seismic performance target of "Near Collapse" level that is defined by EN 1998-3, if $\left(\theta_{yb} + \theta_{pb}\right) < \theta_{ub}$ is true for the examined structural member, then the seismic demand is satisfied, where θ_{ub} is the available ultimate chord rotation of the member.

With reference to (ii) Case, we consider that after the appearance of the first plastic hinge at the beam-end, the second plastic hinge appears at the column bottom-section. For greater lateral relative displacement u_{dem}, the bending moment M_i at joint (i) is equal with the available yielding moment (by Eq.(1)), namely $M_i = M_{yc}$, while the demand column chord rotation $\theta_{ij"}$ is going to greater than available one θ_{yc}, namely $\theta_{ij"} > \theta_{yc}$, FIG.(3). Then, the chord rotation $\theta_{2"}$ is given by $\theta_{2"} = \delta_{c2"}/H_{2"}$, where $Q_{c"}$ is the current shear force of ij column at this step of analysis, $H_{2"} = M_{yb}/Q_{c"}$, $\delta_{c2"} = Q_{c"}/k_{2"}$ and $k_{2"} = 3EI_{ij,eff}/H_{2"}^3$.

Thus, the total (yield plus plastic) beam chord rotation at joint (j), as a function of the seismic demand horizontal relative displacement u_{dem} , is geometrically given by:

$$\theta_{b,tot} = \theta_{ij"} - \theta_{2"} = \frac{u_{dem}}{H_{ij}} - \frac{\delta_{c2"}}{H_{2"}} \tag{12}$$

Therefore, the seismic demand plastic beam rotation θ_{pb} of the first plastic hinge at the beam-end is calculated by:

$$\theta_{pb} = \theta_{b,tot} - \theta_{yb} \quad \Rightarrow$$

$$\theta_{pb} = \left(\frac{u_{dem}}{H_{ij}} - \frac{\delta_{c2"}}{H_{2"}}\right) - \left(\frac{u_{y1}}{H_{ij}} - \frac{\delta_{c2}}{H_2}\right) \tag{13}$$

or

$$\theta_{pb} = \frac{u_{dem} - u_{y1}}{H_{ij}} - \frac{\delta_{c2"}}{H_{2"}} + \frac{\delta_{c2}}{H_2} \tag{14}$$

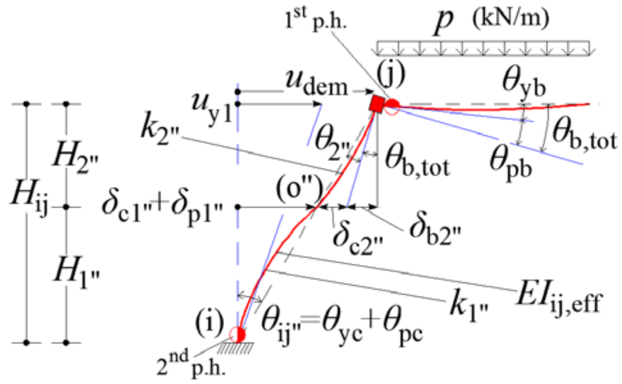

FIG. 3 ADVANCED STEP OF ANALYSIS. PLASTIC HINGES AT BEAM-END AND COLUMN BOTTOM

On the other hand here, the plastic hinge at the column bottom must be checked. For this target (FIG.3), we consider the inelastic ideal cantilever i-o" as part of column ij, where the free-end o" has the sum of the elastic displacement $\delta_{c1"} = Q_{c"}/k_{1"}$ and the unknown plastic displacement $\delta_{p1"}$, while $k_{1"} = 3EI_{ij,eff}/H_{1"}^3$ is the lateral stiffness of the ideal cantilever i-o". Then, for a known u_{dem}, at the column's null-moment point o" , the seismic demand plastic displacement $\delta_{p1"}$ is directly calculated as being:

$$\delta_{p1"} = u_{dem} - \delta_{c1"} - \delta_{c2"} - \delta_{b2"} \tag{15}$$

where $\delta_{b2"} = \theta_{b,tot} \cdot H_{2"}$.

Thus, the seismic demand plastic rotation θ_{pc} of the column plastic hinge is given by:

$$\theta_{pc} = \frac{\delta_{p1"}}{H_{1"}} \tag{16}$$

Afterwards, the checking of the seismic performance matrix can be performed as already above-mentioned in the present paragraph.

Interior Roof Joint of Single-Storey Frame

1) Calculation of Seismic Demand Beam Yielding Chord Rotation

Consider that the available yielding moment M_{yb1} at the beam-end of beam I, that converge at joint (j), is known by Eq.(1), (FIG.4). In addition, we consider known that the horizontal relative displacement $u_j = u_{y1}$, which takes place when the first plastic hinge (with yielding moment M_{yb1}) appears at the end of beam I. Then, the seismic demand column chord rotation θ_{ij} at the column bottom-section is directly given by $\theta_{ij} = u_j / H_{ij}$, which is less than the available column chord rotation θ_{yc} that has been calculated by Eq.(1). Then, at the top of the column, the demand column chord rotation θ_2 can be calculated geometrically by $\theta_2 = \delta_{c2} / H_2$, where M_{cj} & Q_c is the column bending moment & the respective shear force at j-section, $H_2 = M_{cj} / Q_c$, $\delta_{c2} = Q_c / k_2$ and $k_2 = 3EI_{ij,eff} / H_2^3$. Therefore, the seismic demand beam yielding rotation θ_{yb} of the beam-end of beam I is calculated as a function of the horizontal relative displacement u_j of the single-storey frame:

$$\theta_{yb} = \theta_{ij} - \theta_2 = \frac{u_j}{H_{ij}} - \frac{\delta_{c2}}{H_2} \tag{17}$$

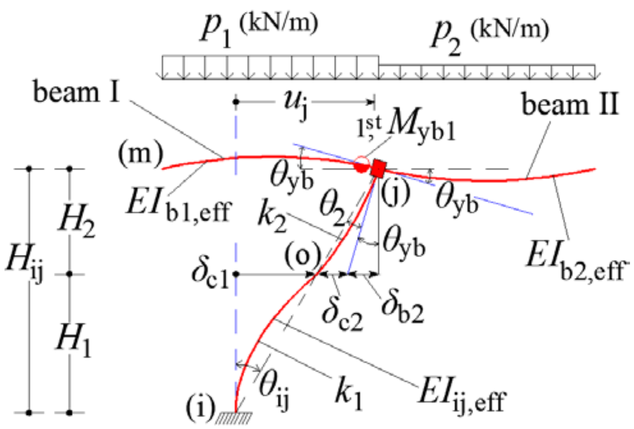

FIG. 4 ESTIMATION OF SEISMIC DEMAND YIELDING BEAM CHORD ROTATION AT INTERIOR ROOF JOINT

2) *Calculation of the Seismic Demand Plastic Beam Chord Rotation*

The plastic rotation θ_{pb}, which takes place in the first plastic hinge, can be calculated for a horizontal relative displacement $u_{dem} > u_{y1}$ (FIG.5). In general, it is known that the seismic demand chord rotation at the column bottom-section is always $\theta_{ij'} = u_{dem} / H_{ij}$. However, at this stage, the main question is this: how can we calculate the demand beam plastic rotation θ_{pb} for $u_{dem} = u'_j$ when $u'_j > u_{y1}$ is true, in the following cases?

- (i) Case: when the second plastic hinge (at beams or columns) has not appeared.
- (ii) Case: when the second plastic hinge has appeared at the end of beam II.
- (iii) Case: when the second plastic hinge has appeared at the column bottom.
- (iv) Case: when the second plastic hinge has appeared at the column top.

If (i) or (ii) cases are validated, then Eqs.(3-11) are directly used; if (iii) case is true then Eqs.(12-16) are directly used; However, if (iv) case is true, then we get FIG.6, where the known demand horizontal relative displacement is u_{y2}, the second plastic hinge appears at the column top-section. In the later case, the seismic demand chord rotation at the column bottom-section is $\theta_{ij''} = u_{y2} / H_{ij}$.

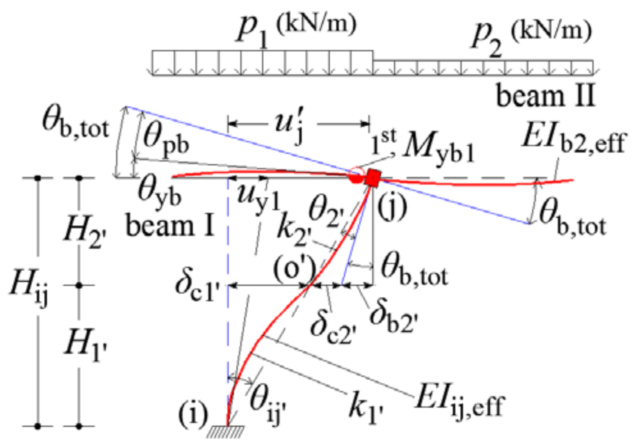

FIG. 5 ESTIMATION OF SEISMIC DEMAND PLASTIC BEAM CHORD ROTATION AT INTERIOR ROOF JOINT

The seismic demand chord rotation $\theta_{2''}$ is geometrically calculated as being $\theta_{2''} = \delta_{c2''} / H_{2''}$, where M_{ycj} & $Q_{c''}$ is the column yielding bending moment & the respective shear force at j-section, $H_{2''} = M_{ycj} / Q_{c''}$, $\delta_{c2''} = Q_{c''} / k_{2''}$ and $k_{2''} = 3EI_{ij,eff} / H_{2''}^3$. Then, the total rotation $\theta_{b,tot}$ of joint (j) is given by:

$$\theta_{b,tot} = \theta_{ij''} - \theta_{2''} = \frac{u_{y2}}{H_{ij}} - \frac{\delta_{c2''}}{H_{2''}} \tag{18}$$

Thus, the seismic demand plastic rotation θ_{pb} of the first plastic hinge at the beam-end is given (FIG.6) by:

$$\theta_{pb} = \theta_{b,tot} - \theta_{yb} \quad \Rightarrow$$

$$\theta_{pb} = \left(\frac{u_{y2}}{H_{ij}} - \frac{\delta_{c2"}}{H_{2"}}\right) - \left(\frac{u_{y1}}{H_{ij}} - \frac{\delta_{c2}}{H_2}\right) \qquad (19)$$

or

$$\theta_{pb} = \frac{u_{y2} - u_{y1}}{H_{ij}} - \frac{\delta_{c2"}}{H_{2"}} + \frac{\delta_{c2}}{H_2} \qquad (20)$$

Afterwards, for a greater horizontal relative displacement $u_{dem} > u_{y2}$, the column does not contribute το joint stiffness, which means that the demand beam plastic rotation θ_{pb} remains constant, without increasing, despite the fact that the horizontal relative displacement u_{dem} is greater, since all additional plastic rotations inserting into the column top-section plastic hinge.

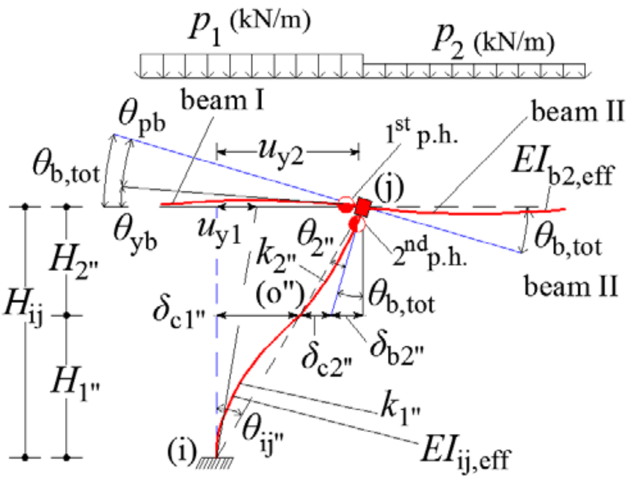

FIG.6 ESTIMATION OF SEISMIC DEMAND PLASTIC BEAM CHORD ROTATION AT INTERIOR ROOF JOINT WITH PLASTIC HINGE AT COLUMN TOP

Exterior and Interior Floor Joint in a Multi-Storey Frame

1) Calculation of Seismic Demand Beam Yielding Chord Rotation

Consider that, just as the 1st plastic hinge appears at the beam-end of *j*-floor, the horizontal floor displacements u_i, u_j and u_k of *i*-, *j*- and *k*-floor, respectively, are known (FIG.7). In this case, the seismic demand "floor angular deformation" θ_{ij} and θ_{jk} for columns ij and jk, respectively, are given by:

$$\theta_{ij} = \frac{u_j - u_i}{H_{ij}} \qquad (21)$$

$$\theta_{jk} = \frac{u_k - u_j}{H_{jk}} \qquad (22)$$

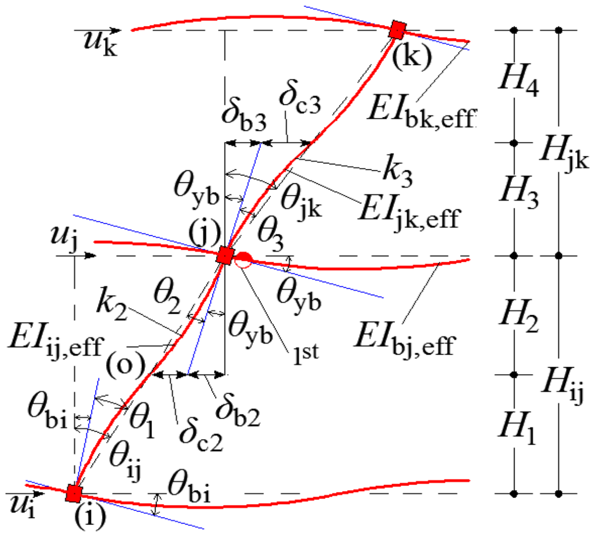

FIG. 7 ESTIMATION OF SEISMIC DEMAND YIELDING BEAM CHORD ROTATION AT INTERIOR FLOOR JOINT

Then, the seismic demand column chord rotation is $\theta_1 = \theta_{ij} - \theta_{bi}$ for the bottom-section of column ij (FIG.7). Moreover, at the top of column ij, its chord rotation is $\theta_2 = \delta_{c2}/H_2$, where M_{c2} & Q_{c2} is the column yielding bending moment & the respective shear force of ij column at j-section, $H_2 = M_{c2}/Q_{c2}$, $\delta_{c2} = Q_{c2}/k_2$ and $k_2 = 3EI_{ij,eff}/H_2^3$ (FIG.7). Moreover, at the base of column jk, its chord rotation is $\theta_3 = \delta_{c3}/H_3$, where M_{c3} & Q_{c3} is the column yielding bending moment & the respective shear force of jk column at j-section, $H_3 = M_{c3}/Q_{c3}$, $\delta_{c3} = Q_{c3}/k_3$ and $k_3 = 3EI_{jk,eff}/H_3^3$. Thus, the seismic, demand yield chord rotation θ_{yb} of the beam-end is given by:

$$\theta_{yb} = \theta_{ij} - \theta_2 = \theta_{jk} - \theta_3 \qquad (23)$$

or more accurately:

$$\theta_{yb} = \frac{(\theta_{ij} - \theta_2) + (\theta_{jk} - \theta_3)}{2} \qquad (24)$$

2) Calculation of Seismic Demand Beam Plastic Chord Rotation

After the appearance of the 1st plastic hinge, we consider that the r/c frame presents larger seismic demand horizontal floor displacements u'_i, u'_j and u'_k of the *i*-, *j*- and *k*-floor, respectively (FIG.8), while the 2nd plastic hinge appears somewhere, let us say, at the base of column jk.

The seismic demand "floor angular deformation" $\theta_{ij'}$ and $\theta_{jk'}$ of columns ij and jk, respectively, are given by:

$$\theta_{ij'} = \frac{u'_j - u'_i}{H_{ij}} \tag{25}$$

$$\theta_{jk'} = \frac{u'_k - u'_j}{H_{jk}} \tag{26}$$

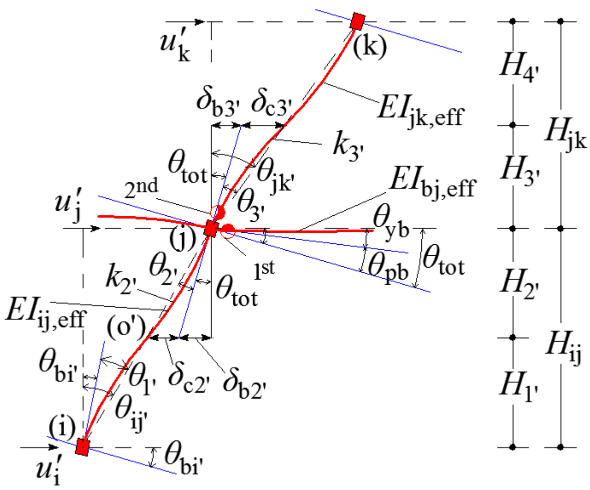

FIG. 8 ESTIMATION OF SEISMIC DEMAND PLASTIC BEAM CHORD ROTATION AT INTERIOR FLOOR JOINT

Then, the seismic demand column chord rotation is $\theta_{1'} = \theta_{ij'} - \theta_{bi'}$ for the bottom-section of column ij (FIG.8). Moreover, at the top of column ij, its chord rotation is $\theta_{2'} = \delta_{c2'}/H_{2'}$, where $M_{c2'}$ & $Q_{c2'}$ is the column yielding bending moment & the respective shear force of ij column at j-section, $H_{2'} = M_{c2'}/Q_{c2'}$, $\delta_{c2'} = Q_{c2'}/k_{2'}$ and $k_{2'} = 3EI_{ij,eff}/H_{2'}^3$. Moreover, at the base of column jk, its demand chord rotation is $\theta_{3'} = \delta_{c3'}/H_{3'}$, where, $H_{3'} = M_{yc3'}/Q_{c3'}$, $\delta_{c3'} = Q_{c3'}/k_{3'}$ and $k_{3'} = 3EI_{jk,eff}/H_{3'}^3$. The total seismic demand rotation θ_{tot} of joint (j) is given by:

$$\theta_{tot} = \theta_{ij'} - \theta_{2'} = \theta_{jk'} - \theta_{3'} \tag{27}$$

or more details:

$$\theta_{tot} = \frac{(\theta_{ij'} - \theta_{2'}) + (\theta_{jk'} - \theta_{3'})}{2} \tag{28}$$

Thus, just as the 2nd plastic hinge appears, the plastic beam-chord rotation θ_{pb} of the 1st plastic hinge is geometrically calculated:

$$\theta_{pb} = \theta_{tot} - \theta_{yb} \quad \Rightarrow$$

$$\theta_{pb} = \frac{(\theta_{ij'} - \theta_{2'}) + (\theta_{jk'} - \theta_{3'})}{2} - \frac{(\theta_{ij} - \theta_{2}) + (\theta_{jk} - \theta_{3})}{2} \tag{29}$$

It is worth noting that each case of plastic hinges is reduced to one of the above-mentioned known cases.

Conclusions

In the present article, the issue of the estimation of the seismic demand chord rotations of structural r/c members, without using inelastic springs at their ends, has been analytically presented according to Eurocode EN 1998-3. In order to estimate the seismic demand chord rotations of the r/c members, geometry in combination the Mechanics Principles have been used, only. All necessary cases have been examined, while all the rest cases reduce to examined ones. This paper can help editors of software relative to structural non-linear analysis (such as non-linear response history analysis or pushover one) of the structures, since with this way the calculation cost of nonlinear analyses of the structures is reduced.

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